A closer look at tumbling toast

M. E. Bacon, George Heald, and Matt James
Department of Physics, Thiel College, 75 College Avenue, Greenville, Pennsylvania 16125

(Received 14 February 2000; accepted 31 May 2000)

The study of the mechanics of tumbling toast provides an informative and entertaining project for undergraduates. The relatively recent introduction of software packages to facilitate the analysis of video recordings, and the numerical solution of complex differential equations, makes such a study an attractive candidate for inclusion in an experimental physics course at the undergraduate level. In the study reported here it is found that the experimentally determined free fall angular velocity of a board, tumbling off the edge of a table, can only be predicted at all accurately if slipping is taken into account. The size and shape of the board used in the calculations and in the experiments were roughly the same as that of a piece of toast. In addition, it is found that the board, tumbling from a standard table of height 76 cm, will land butter-side down (neglecting any bounce) for two ranges of overhang (δ_0). δ_0 is defined as the initial distance from the table edge to a vertical line drawn through the center of mass when the board is horizontal. For our board (length 10.2 cm) the approximate ranges of overhang are 0–0.8 and 2.7–5.1 cm. The importance of the 0–0.8 cm (only 2% of all possible overhangs for which tumbling is possible) favoring a butter-side down landing should not be underestimated when pondering the widely held belief that toast, tumbling from a table, usually falls butter-side down. © *2001 American Association of Physics Teachers.*
[DOI: 10.1119/1.1289213]

I. INTRODUCTION

Over the past five years a number of papers dealing with the mechanics of tumbling toast have appeared in the literature.[1–5] These interesting and entertaining papers were motivated, in part, by a desire to verify and explain the popularly held belief that toast, falling from a table, usually lands butter-side down.

A simplified theoretical treatment of the dynamics of tumbling toast has been given by Matthews,[1] and independently by Steinert.[3] These treatments assume that the toast has negligible thickness and that the toast leaves the table once the maximum static friction force is exceeded. With these assumptions, the free fall angular velocity can be determined analytically [see Eqs. (14) and (15) below]. For a range of coefficients of static friction (0.2–0.6) it was found that the free fall angular velocities of tumbling toast (see Table II of Ref. 3) were such that the toast would rotate through angles less than 270° when falling from a table of height 76 cm (see Table III of Ref. 3). This means that the toast lands butter-side down. However, for small overhangs the angle of rotation is often less than 90°, implying a butter-side up landing. This is not shown in Table III of Ref. 3 but can be easily verified (see our Fig. 8). On the other hand, Matthews, using the same theoretical framework as Steinert, argues that the toast falls butter-side down for small overhangs.

Experimentally, Stevenson and Bacon[5] found that there seemed to be agreement between the angular velocity calculated from measurements of the angle at which the toast appears to leave the table [see Eq. (14) below], and the experimentally measured free fall angular velocity. However, the magnitude of the angular velocity was significantly larger than one would expect on the basis of the theory developed by Matthews and Steinert, assuming coefficients of static friction in the range 0.2–0.6. Clearly there are a number of inconsistencies in the studies carried out heretofore.

Until a few years ago, further experimental and theoretical investigation of tumbling toast might have been tedious and hardly worth the effort. However with the availability of video analysis software packages such as VIDEOPOINT[6] to aid the experimental investigation, and the availability of sophisticated modeling programs such as STELLA[7] to facilitate the numerical solution of nonlinear differential equations with a minimum of programming effort, the detailed study of tumbling toast becomes a viable and interesting project for undergraduates in an experimental physics course.

In the present paper we report on measurements of the coefficients of kinetic and static friction for a board, of roughly the same dimensions as a piece of toast, and present theoretical calculations of the expected angular velocity of free fall using a theoretical framework which includes slipping. The theoretical free fall angular velocities are compared to previous calculations and to experimental results obtained from video recordings of a tumbling board. Finally, the total angle of rotation, during free fall from a table of height 76 cm, is computed for various overhangs, and the results compared with observations of butter-side down and butter-side up landings. Although the coefficients of friction for a piece of toast and the board we have used can be quite different, theory indicates that the general behavior exhibited by board and toast is quite similar. Suggestions for additional projects involving tumbling objects are given in the final section.

II. THEORY

A common bread in Greenville, PA is Nickles jumbo wheat. This bread is made by the Alfred Nickles bakery in Navarre, OH. According to the label on the packaging the Alfred Nickles bakery has been producing ‘‘quality baked goods for a century.’’ A slice of this fine bread measures roughly $10\times 9.5\times 1.3$ cm. Some initial experiments[5] were performed with toast made from this bread, but the unevenness of the surfaces, the crumbly nature of the toast, variations from slice to slice, and its tendency to become hard and brittle over time affects the reproducibility of the experiments. Consequently, a plywood board, of comparable di-

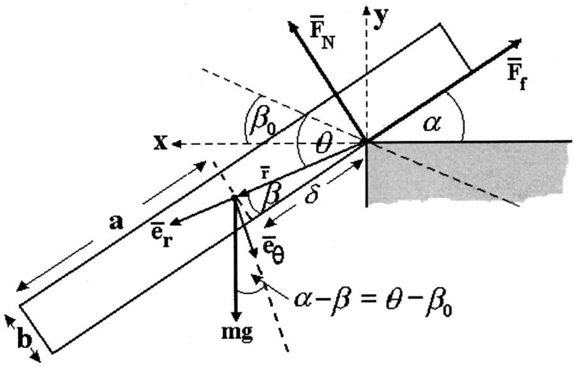

Fig. 1. Coordinate system (r,θ) and geometry for the tumbling board.

mensions $(10.2\times 10.8\times 1.3\,\mathrm{cm})$, was used for the detailed study presented in this paper. The term board and toast will be used interchangeably in this paper.

Consider a board of half length a and thickness b placed on a horizontal surface (such as a table) so that the center of mass of the board extends past the edge of the surface. The initial overhang (δ_0) is defined to be the horizontal distance from the edge of the table to a vertical line drawn through the center of mass when the board is horizontal (in the case of the board, this is the geometric center). For $0<\delta_0<a$, the board will tumble from the horizontal surface since its center of gravity overhangs the edge of the table.

Figure 1 shows the free body diagram, and the polar coordinate system (r,θ), used to theoretically analyze the dynamics of tumbling toast.

The dynamics of the tumbling board (assumed to be a rigid body) is governed by: (a) Newton's second law applied to the center of mass:

$$\sum \mathbf{F}_{\text{ext}}=m\mathbf{a}_{\text{c.m.}}, \tag{1}$$

and (b) the torque-angular momentum equation:

$$\sum \boldsymbol{\tau}_{\text{c.m.}}=d\mathbf{J}_{\text{c.m.}}/dt. \tag{2}$$

For the case of tumbling toast shown in Fig. 1, the angular momentum $(\mathbf{J}_{\text{c.m.}})$ can be written as

$$\mathbf{J}_{\text{c.m.}}=I_{\text{c.m.}}\dot{\boldsymbol{\alpha}}. \tag{3}$$

Applying Eq. (1) to Fig. 1 yields

$$F_N\sin\beta-F_f\cos\beta+mg\sin(\theta-\beta_0)=m(\ddot{r}-r\dot{\theta}^2), \tag{4}$$

$$-F_N\cos\beta-F_f\sin\beta+mg\cos(\theta-\beta_0)=m(r\ddot{\theta}+2\dot{r}\dot{\theta}). \tag{5}$$

Applying Eq. (2), with $\mathbf{J}_{\text{c.m.}}$ given by (3), gives

$$(F_N\cos\beta)r+F_f b/2=m(a^2/3+b^2/12)\ddot{\alpha}. \tag{6}$$

α, β, and θ are related through $\alpha=\theta-\beta_0+\beta$. β_0 is constant and is given by $\tan\beta_0=b/(2\delta_0)$. Therefore, $\ddot{\alpha}=\ddot{\theta}+\ddot{\beta}$ and Eq. (6) can be written

$$(F_N\cos\beta)r+F_f b/2=m(a^2/3+b^2/12)(\ddot{\theta}+\ddot{\beta}). \tag{7}$$

In these equations, $r=\sqrt{(b^2/4+\delta^2)}$, $\sin\beta=b/(2r)$, and $\cos\beta=\delta/r$. Using $\sin\beta=b/(2r)$, it can be shown that

$$\dot{\beta}=-\dot{r}b/(2r^2\cos\beta) \tag{8}$$

and

$$\ddot{\beta}=\dot{r}^2\sin\beta(1+\cos^2\beta)/(r^2\cos^3\beta)-\ddot{r}\tan\beta/r. \tag{9}$$

Equations (4), (5), and (7)–(9) are the most general equations describing the dynamics of tumbling toast.

Initially, as the toast begins to rotate about an axis along the edge of the table, there is no slipping. Hence, $r=r_0$ and $\beta=\beta_0$ are constants, and $\alpha=\theta$. For this initial situation, therefore, Eqs. (4), (5), and (6) become

$$F_N\sin\beta_0-F_f\cos\beta_0+mg\sin(\theta-\beta_0)=-mr_0\dot{\theta}^2, \tag{10}$$

$$-F_N\cos\beta_0-F_f\sin\beta_0+mg\cos(\theta-\beta_0)=mr_0\ddot{\theta}, \tag{11}$$

$$(F_N\cos\beta_0)r_0+F_f b/2=m(a^2/3+b^2/12)\ddot{\theta}. \tag{12}$$

r_0 is constant and is given by $r_0=\sqrt{(b^2/4+\delta_0^2)}$, where δ_0 is the initial overhang; $\sin\beta_0=b/(2r_0)$ and $\cos\beta_0=\delta_0/r_0$ are also constant.

Combining (11) and (12) yields

$$\ddot{\theta}=(r_0 g\cos(\theta-\beta_0))/((a^2+b^2)/3+\delta_0^2). \tag{13}$$

The equations for thin toast, under no-slip conditions, developed by Matthews[1] and Steinert,[3] can be obtained from Eqs. (10), (11), and (13) by setting $b=0$, $\beta_0=0$, and $r_0=\delta_0$. The equations can then be solved for angular velocity $(\dot{\theta}=\dot{\alpha})$ and for the ratio F_f/F_N to yield[1,3]

$$\dot{\alpha}^2=(6g\delta_0\sin\alpha)/(a^2+3\delta_0^2), \tag{14}$$

$$F_f/F_N=(1+9\delta_0^2/a^2)\tan\alpha. \tag{15}$$

The toast begins to slip when F_f/F_N becomes equal to the coefficient of static friction. Steinert,[3] using a range of coefficients of static friction, calculated critical values of α from (15). The critical value of α is the angle that the toast makes with the horizontal at the moment of slipping. The angular velocity at these angles (the critical angular velocity) was then calculated using Eq. (14). The free fall angular velocities were assumed by Steinert to be equal to these angular velocities. This implicitly assumes that the toast leaves the table once slipping starts (see Tables I–III of Ref. 3).

In the experiments performed by Stevenson and Bacon,[5] a tumbling board and a piece of tumbling toast were recorded on videotape at 30 frames per second. The experimentally determined free fall angular velocities were found to be significantly greater than any predicted from reasonable coefficients of static friction (0.2–0.6), but tended to agree with those calculated from Eq. (14) using measurements of the angle at which the toast appeared to leave the table. However, the precise time and angle at which the board leaves the table are difficult to measure because of insufficient time resolution (1/30 s).

In Ref. 5 it was suggested that the larger than expected free fall angular velocities may be due to assumptions regarding the use of the standard expression for static friction $(F_f\leqslant\mu_s F_N)$ and/or due to the neglect of any sliding motion. In addition to these two factors it was suggested that the thickness of the board may also play a role. As we shall see, the latter effect is of minor importance.

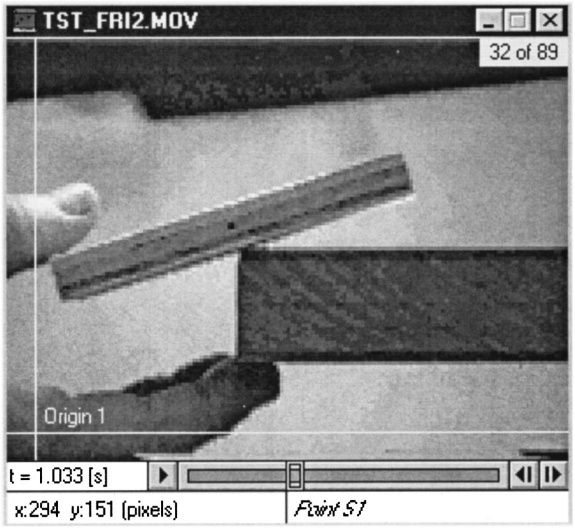

Fig. 2. Typical video frame used to measure the coefficient of static friction for a tumbling board in contact with an edge. The thumb is used to control the gentle tilting of the board. Generally the video clip contains a number of slipping frames as the experimenter gets a feel for controlling the board.

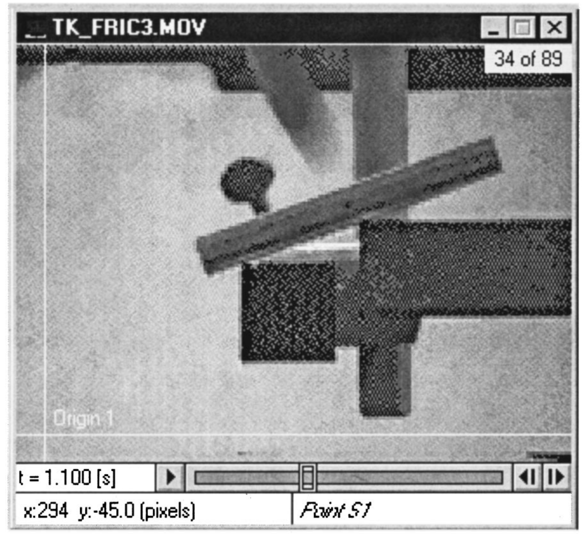

Fig. 3. Typical video frame from a video used to measure the coefficient of kinetic friction of a board in contact with an edge.

III. EXPERIMENTAL DETERMINATION OF THE COEFFICIENTS OF STATIC AND KINETIC FRICTION

A. Static friction

The coefficient of static friction was determined by videotaping the board while it was gently tilted and began to slide. Then VIDEOPOINT was used to measure the angle at which slipping began. Figure 2 shows a typical video frame at the time of slipping. The angle that the plane of the board makes with the surface of the table is gently increased using the thumb to control and push down on the board. Without the use of the video it is almost impossible to obtain a reliable measure of the angle.

Application of Newton's second law with $\mathbf{a}_{\text{c.m.}}=0$, and using the condition that $F_f=\mu_s F_N$ at the point of slipping, leads to $\mu_s=\tan\alpha_c$. The angle α_c is the angle at which slipping begins. Repeated measurement of α_c yielded a value for $\mu_s=0.32\pm 0.02$.

It should be noted that in measurements made by Matthews,[1] the coefficients of static friction for bread and toast were measured by simply tilting the surface upon which the toast was placed. We have found, however, that the coefficient of static friction measured in this way can be quite different from that measured for an edge. For example, for our board the coefficient of static friction for flat surface contact was found to be about 0.4 as opposed to 0.32 for the edge. On the other hand, for our Nickles toast, the coefficient of static friction was found to be 0.58 for the flat surface and 0.62 for the edge. Hence, for the Nickles toast the edge friction is larger than that for a flat surface, while the reverse is true for the board. In addition, we note that the coefficient of static friction for Nickles toast is significantly higher than that for the toast used by Matthews ($\mu=0.25$). For any quantitative work it is therefore important to measure the coefficients of friction for the tumbling object and the edge one is going to use.

B. Kinetic friction

The coefficient of kinetic friction was measured by recording the motion of the board sliding on two edges of the same table material as shown in Fig. 3.

By applying Newton's second law to this situation and using the relationship $F_f=\mu_k F_N$, it is easy to show that

$$\mu_k=(g\sin\alpha-a_{\text{c.m.}})/(g\cos\alpha).$$

α is the angle the board makes with the horizontal, and $a_{\text{c.m.}}$ is the acceleration of the center of mass.

Using VIDEOPOINT the position of the board can be measured as a function of time. A plot of these data as a function of time for one value of α is shown in Fig. 4. Also shown is a fit of the data to the function $a_{\text{c.m.}}t^2/2$ using PSIPLOT.[8] This yielded the acceleration $a_{\text{c.m.}}$. Using different angles, the value of the coefficient of kinetic friction was determined to be $\mu_k=0.24\pm 0.02$.

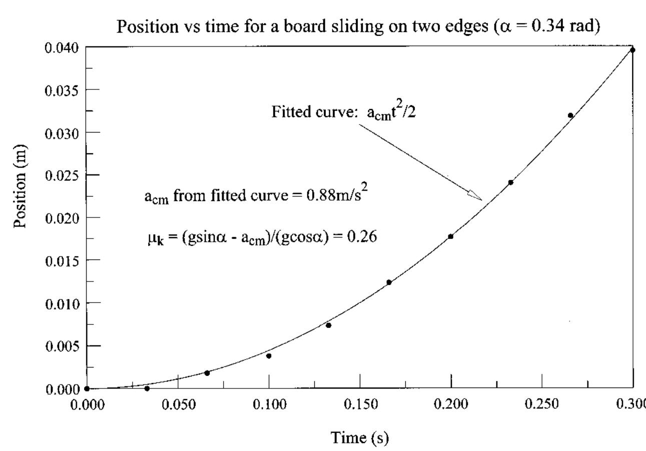

Fig. 4. Position vs time graph obtained from video (Fig. 3).

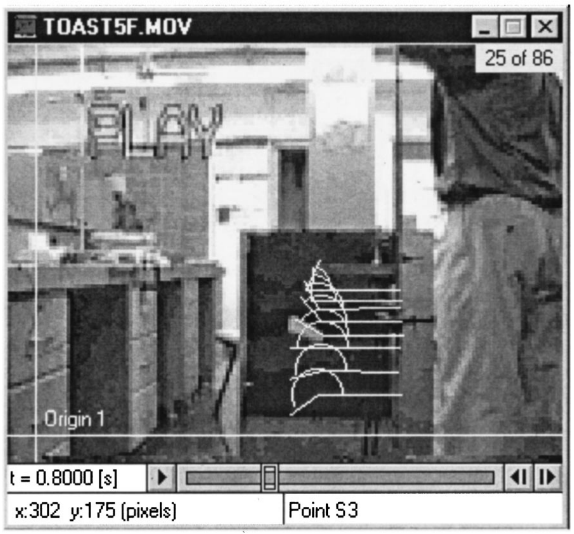

Fig. 5. Experimental setup for measuring the free fall angular velocity as a function of overhang.

IV. MEASUREMENT OF THE ANGULAR VELOCITY OF A TUMBLING BOARD

The motion of our tumbling board, after it left the table, was recorded at 30 frames per second for different values of overhang. The video recordings were then analyzed using VIDEOPOINT, and the angle alpha as a function of time was determined. Figure 5 shows a frame of the video (for an overhang of 1.1 cm) with a typical set of angular measurements superimposed. The angular measurements were made after the board left the table.

Figure 6 shows a plot of the measured angle versus time, and a fit of the data to a straight line. The slope of the line is the free fall angular velocity.

Table I gives the various overhangs, the corresponding measured average angular velocities, and their standard deviations. These data are shown plotted in Fig. 7 along with the theoretical predictions.

The solid lines are the theoretical results for a board with thickness 1.3 cm and the dashed lines are for a board with zero thickness. To obtain these curves, Eqs. (10), (11), and

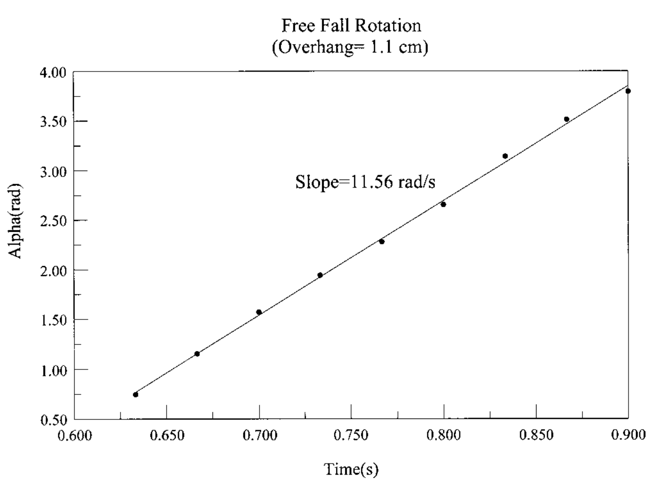

Fig. 6. Typical data (closed dots) of angle of rotation vs time during free fall. The solid line is a straight line fit to the data.

Table I. Experimentally determined angular velocities for different overhangs.

Overhang (cm)	Measured angular velocity (rad/s)
0.10	6.93 ± 0.26
0.30	8.80 ± 0.24
0.55	10.03 ± 0.34
0.85	10.90 ± 0.36
1.10	11.57 ± 0.37
1.30	12.2 ± 0.67
1.60	12.47 ± 0.62
1.85	12.57 ± 0.37
2.10	12.17 ± 0.31
2.30	12.43 ± 0.55

(13) were first solved numerically for $\dot{\theta}$ and θ using STELLA. An analytic solution for $\dot{\theta}$ is possible but the solution for θ involves an elliptic integral. The ratio F_f/F_N was also calculated using these equations, and the values of $\dot{\theta}$ and θ, for $F_f/F_N = 0.32$, were determined. These are the values of $\dot{\theta}$ and θ at which slipping begins. With these values, $r = r_0$, $\dot{r} = 0$ as initial conditions, and a coefficient of kinetic friction $\mu_k = 0.24$, Eqs. (4), (5), (7), and (9) were then solved numerically for $\dot{\theta}$, θ, $\dot{r}$, r, and $\dot{\beta}$. The angular velocity, $\dot{\alpha} = \dot{\theta} + \dot{\beta}$, when the normal force (F_N) became zero was then determined. This is the free fall angular velocity. As can be seen from Fig. 7, the agreement between theory and experiment, over the range of overhangs considered, is excellent. The theoretical curves for thin toast were calculated in a similar fashion. The STELLA models can be viewed at *http://www.thiel.edu/academics/physics/projects/toast/default.htm.*

From Fig. 7, we can conclude that slipping plays an essential role in the dynamics of tumbling toast, and must be considered in order to get agreement with the measured angular velocities. Previous theory (Matthews[1] and Steinert[3]) predicts the lower solid curve, which is in complete disagree-

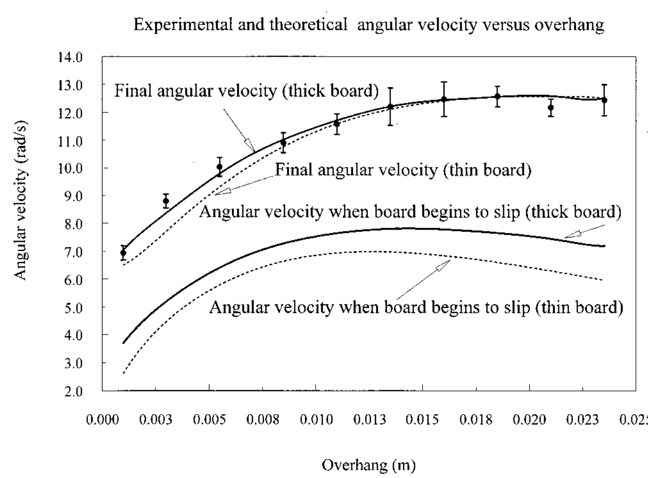

Fig. 7. Experimental angular velocity (closed dots with error bars) vs overhang for a tumbling board (length $2a = 10.2$ cm and thickness $b = 1.3$ cm. $\mu_s = 0.32, \mu_k = 0.24$). Solid lines are the theoretical curves obtained by numerically solving the differential equations for the case of a thick board. Dashed lines are the theoretical results for a board with zero thickness. The lower two curves are the curves obtained using the simple theory which ignores slipping.

Table II. Observed butter-side up or butter-side down behavior of a tumbling board.

Overhang (cm) ±0.02 cm	Board lands butter-side
0.10	**Down**
0.30	**Down**
0.63	**Down**
0.79	**Down**
0.81	Up
0.85	Up
1.10	Up
1.30	Up
1.60	Up
1.85	Up
2.10	Up
2.35	Up
2.80	Up
3.03	**Down**
3.28	**Down**
3.40	**Down**
3.7	**Down**

ment with experiment. In addition, Fig. 7 shows that thickness plays a rather minor role in determining the final angular velocity.

V. BUTTER-SIDE UP OR BUTTER-SIDE DOWN?

The butter-side up or butter-side down behavior of our board was investigated by varying the overhang and observing whether the board came to rest with its butter side up or down in a sandbox positioned underneath the edge of the table. The use of the sandbox prevented any bounce. The table surface was adjusted so that it was 76 cm above the surface of the sand. Table II shows the experimental results.

Clearly this behavior is quite different from the behavior one might expect from the data given in Table III of Ref. 3. According to that table, a board with coefficient of static friction around 0.3 should land butter-side down for all but the largest overhangs. It should also be noted that the calculations show that the board should land butter-side up for overhangs close to zero (see our Fig. 8) since the board rotates through less than 90° for the smallest overhangs.

The STELLA modeling program can easily be adapted to determine the butter up–butter down behavior of the board. Referring to Fig. 1, where the positive y axis is taken to be vertically up and the origin is at the table edge, the y coordinate of the center of mass is given by

$$y = -r\sin(\theta - \beta_0).$$

Differentiating with respect to time yields the y component of the center-of-mass velocity,

$$\dot{y} = -\dot{r}\sin(\theta - \beta_0) - r\cos(\theta - \beta_0)\,\dot{\theta}.$$

The values of r, $\dot{r}$, θ, and $\dot{\theta}$, as the board breaks contact with the edge, can be obtained from the STELLA modeling program as a function of overhang. Hence the initial values (y_0 and $\dot{y}_0$) for the y coordinate and y component of the velocity for the free falling center of mass can be determined. The angle between the surface of the board and the horizontal as the board leaves the edge (α_0), and the corresponding angular velocity ($\dot{\alpha}_0$) (the free fall angular velocity), can also be obtained from the STELLA model. Using these values, the y coordinate of the edge of the board nearest the floor at any instant of time can be calculated using

$$y(t)_{\text{nearest edge}} = y_0 + \dot{y}_0 t - 4.9t^2 - a|\sin(\alpha_0 + \dot{\alpha}_0 t)|.$$

From this equation, the time (t_f) and the corresponding total angle of rotation $\alpha_T = (\alpha_0 + \dot{\alpha}_0 t_f)$ for $y_{\text{nearest edge}} = -76$ cm can be calculated. The board will land butter-side down for $90° < \alpha_T < 270°$; otherwise it will land butter-side up. The calculated α_T is shown plotted as a function of overhang in Fig. 8 (solid line). The dashed curve is the curve generated using the computational procedure of Ref. 3. Comparison of Fig. 8 and Table II shows that our theoretical calculations predict the observed behavior very well, while calculations which ignore slipping (dashed curve) do not.

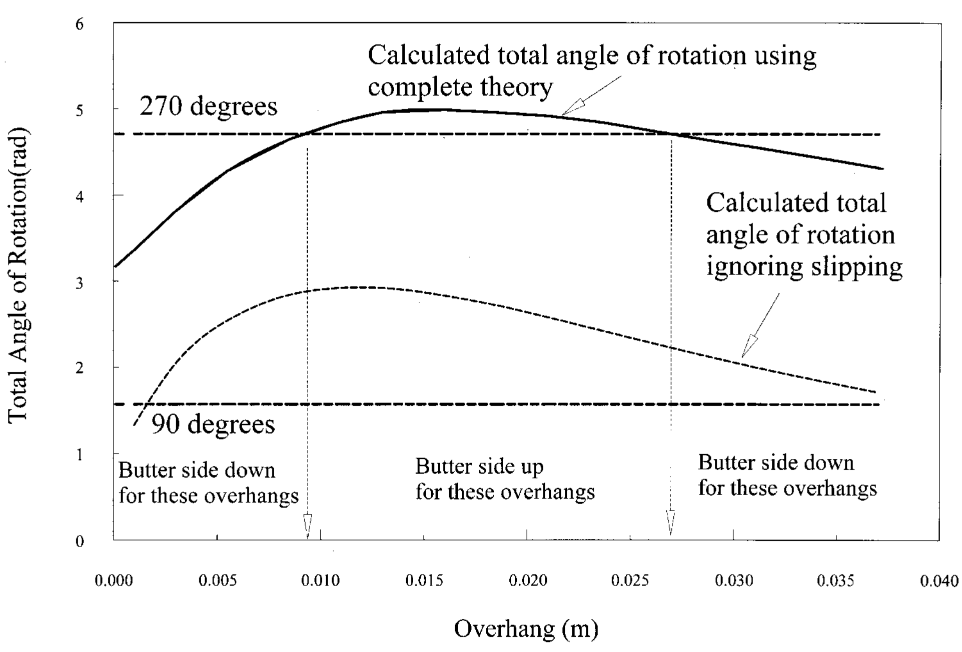

Fig. 8. Theoretical calculations of total angle of rotation for board (length $2a = 10.2$ cm and thickness $b = 1.3$ cm, $\mu_s = 0.32$, $\mu_k = 0.24$), falling from a surface 76 cm above the floor. The board lands butter-side down for angles of rotation between 90° and 270°.

VI. CONCLUSIONS AND SUGGESTED TOPICS FOR FURTHER STUDY

In order to obtain an accurate description of the behavior of tumbling toast, the slipping regime must be included. When this is done good agreement between the theoretical free fall angular velocity and the experimentally observed angular velocity is obtained. Consequently, the butter-side up or down behavior of a tumbling board can only be explained if slipping is taken into account.

As can be seen from Fig. 8 and Table II, the board used in the study reported here lands butter-side down, when falling from a table 76 cm high, for overhangs given by

$$0<\delta_0<0.8 \text{ cm}, \quad 2.7>\delta_0<5.1 \text{ cm}.$$

These two ranges constitute about 63% of the complete overhang range. At first glance this might seem somewhat discouraging (for butter-side downers) since it would be nice if the board almost always fell butter-side down as is predicted by calculations which ignore slipping. Unfortunately, this is not the case. However, the importance of the 0–0.8 cm range of overhangs should not be underestimated, even though it constitutes only about 2% of the overhang range for which tumbling is possible. It is important because this overhang range has to occur in almost all plausible accident scenarios (e.g., the toast is carelessly bumped off a table top) before tumbling takes place. If toast invariably lands butter-side down when it is accidentally displaced from a table, as popular opinion would have us believe, then the ''usual conditions'' (initial speed and horizontal position) of the toast as it leaves the table needs to be established and shown to be consistent with a small overhang at low speed (see also Ref. 1).

For those who might be interested in pursuing this vital subject further we have a number of suggestions for further study.

(1) The STELLA modeling program can be easily adapted to study the dynamics of a board that slides toward the edge with varying speeds before tumbling. This could also be fairly easy to set up experimentally using a ramp.

(2) Asymmetric orientations of the toast could also be studied theoretically and experimentally. Clearly the accidental displacement of toast does not dictate that the toast arrive at the edge in a nice symmetric fashion.

(3) What about bagels? Do bagels fall cream-cheese-side down? Does the universe favor bagels? The nice thing about a bagel, besides the hole in the middle, is its symmetry.

(4) Is there a way to redesign the edge of a table or plate to ensure that toast will almost always rotate through more than 270° (or less than 90°) and hence never fall butter-side down?

(5) Perhaps the shape of a loaf of bread can be changed so that toast made from that bread never lands butter-side down. How would toast shaped like a dog biscuit, a pierogie, or a triangle behave?

Finally we note that, during the present investigation, attempts were made to make a comparison between the experimentally measured α's and r's while the board was in contact with the table, and the values predicted theoretically, by videotaping the board from close up. This met with mixed success because of the inadequate time resolution (30 frames/s) and unknown shutter speed of the video recording. In most cases, however, we were able to confirm a ballpark time for the onset of slipping and a time and angle for loss of contact with the edge consistent with that expected from the theory and consistent with previous measurements.[5]

If the frame capture speed can be increased (to 100–200 frames/s) it would be interesting to carry out a detailed comparison between the theoretically predicted and experimentally measured α's and r's. This is a particularly intriguing prospect since the STELLA simulation (with a thick board) indicates that at the start of the motion the static friction force must point outwards away from the edge. As time goes on the static friction becomes zero and then pulls inwards, reaching its maximum static value roughly midway through the motion. In the simulations run to date, the assumption was then made that the friction force drops immediately to its kinetic value ($\mu_k F_N$) and remains there for the remainder of the time that the board is in contact with the edge. Perhaps with improved time resolution, detailed studies could yield information on the complex behavior of friction and in particular the velocity weakening and strengthening of kinetic friction.[9,10] In this regard it should be noted that the STELLA modeling program allows for the inputting of variations in μ_k in the form of a graph.

ACKNOWLEDGMENTS

Purchase of the STELLA software was supported by NSF-ILI Grant No. 9751120. The grant was used to develop computer-based laboratories for a two-semester interdisciplinary course titled *Science and our Global Heritage*. The investigations reported in this paper are not part of this course, but represent a valuable spin-off into other science courses.

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